

Radiation effects on particle's trajectory in the linear level

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Abstract

In this work, we first obtain the linear form of the scalar self-force and then, the effect of self-force on the particle's trajectory is considered. In de Sitter space-time, within the classical approach, we consider this effect. Finally, some limits for the problem are presented.

1 Introduction

In many cases of physical interest, for example in cosmology, radiation is an important subject, since almost all our measurable knowledge of universe is based on radiation. It is well known that radiation of an accelerated charged particle affects the motion of the particle which is known as the Abraham-Lorentz force [1]. The Abraham-Lorentz force is the recoil force that acts back on the radiating particle. It is also called the radiation reaction force. This force carries momentum and is proportional to the square of the charge and also the rate of change in acceleration or the "jerk". Dirac [2] employed space-like geodesics to generalize the Abraham-Lorentz force to the relativistic velocities. Hence, this self-force which is due to the particles own electromagnetic field on itself is called the Abraham-Lorentz-Dirac force. Moreover, in physics we deal with particle as a point-like object. Then an immediate question arises: What happens to the particle's energy on exactly the particle's position? clearly it diverges at the particle's world line. Such difficulty is discussed and resolved by the mean of self-energy [3], and one can find a lot of papers on these subjects (for further review see [4] and refs there in).

In the present study, we limit our analysis to the self-force effect upon the trajectory of an extended charged particle in the linear level in flat background and also in non-relativistic limit in de Sitter space-time. We choose the de Sitter space-time, because according to the cosmological data, this model could well describe our universe with a small non-vanishing cosmological constant [5]. In this space-time, we obtain the geodesic equations for an extended charged particle. Then by solving these equations, the relation for the particle's velocity with respect to time is obtained. This relation will be used to derive the radiation reaction for a non-relativistic charged particle, falling freely in a gravitational field defined by de Sitter geometry.

The organization of this paper is as follows: in section two, we review the self-force, and then we obtain the linear form of the scalar self-force. After that, in flat background, the equation of motion is analyzed. In section 3, after reviewing de Sitter space-time, the geodesics for a freely falling particle in this space-time is studied and as it is shown one can find a gravitational force acting on this particle in local consideration. In this space-time, we find the corrections in the geodesics due to the self-force in the non-relativistic limit.

In this paper for the sake of simplicity, we set $c = 1$.

2 Self-force

The self-force on a charge arises from the interaction of the charge with its own retarded field. The resultant acceleration is proportional to q^2/m and the covariant form of the self-force is given by [6]

$$F_{self}^{\mu} = F_{Sch}^{\mu} + F_{rad}^{\mu}, \quad (2.1)$$

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where F_{Sch}^μ and F_{rad}^μ are the Schott and radiation reaction terms respectively which are defined by:

$$F_{Sch}^\mu = \frac{2}{3}q^2\ddot{u}^\mu, \quad F_{rad}^\mu = \frac{2}{3}q^2u^\mu\dot{u}^\lambda\dot{u}_\lambda, \quad (2.2)$$

where dot stands for the time derivative. Thus the Lorentz-Dirac equation reads:

$$ma^\mu = F_{ext}^\mu + F_{self}^\mu. \quad (2.3)$$

This equation is a third-order differential equation and hence in the consideration of the equation of motion, some difficulties arise, for example, the appearance of the runaway solutions. It means, with a constant applied force the acceleration grows exponentially with time [3, 4]. Some authors explore the quantum mechanical response to these problems [7], but Landau-Lifshitz's remedy seems to work well, in which they considered q^2/m as a very small quantity. This condition means that the radiation reaction must be very small in comparison with the external force that acts on the charge [4, 8, 9], so we have:

$$ma^\mu = F_{ext}^\mu + \frac{2}{3}\frac{q^2}{m}(\delta_\lambda^\mu + u^\mu u_\lambda)\partial_\gamma F_{ext}^\lambda u^\gamma, \quad (2.4)$$

and its non-relativistic limit is as follows [4, 9]:

$$ma = f_{ext} + \frac{2}{3}\frac{q^2}{m}\dot{f}_{ext}. \quad (2.5)$$

The above formulas are in the flat geometry and the analysis of the electromagnetic and gravitational self-forces in curved space-time were first considered in [10] and [11], respectively. But what we are going to consider in this paper is the self-force acting on a particle with scalar charge q in curved space-time with the following equation of motion [12]:

$$m\frac{D}{d\tau}u^\mu = f_{ext}^\mu + \frac{1}{3}\frac{q^2}{m}(\delta_\nu^\mu + u^\mu u_\nu)\dot{f}_{ext}^\nu + \frac{1}{6}q^2(R_\nu^\mu u^\nu + u^\mu R_{\nu\gamma}u^\nu u^\gamma) + f_{self}^\mu, \quad (2.6)$$

where $\frac{D}{d\tau}u_\mu = u^\nu\nabla_\nu u_\mu$ defines the acceleration, $R_{\mu\nu}$ is the Ricci tensor and the self force is given by

$$f_{self}^\mu = q^2 \int_{-\infty}^{\tau} \left(\partial^\nu G_R(x, x') + u^\mu u_\nu \partial^\nu G_R(x, x') \right) d\tau', \quad (2.7)$$

$G_R(x, x')$ is the retarded scalar Green's function in that space-time. This is the simplest generalization of the Eq. (2.3) and it recovers Eq. (2.4) in the flat space-time [13]. By the scalar charge, it means that the charge q is constant along the world line of the particle, note that in contrast to the electromagnetic case, the Klein-Gordon equation does not require conservation of charge [12].

2.1 Self-force at the linear level

At this stage let us consider Eq. (2.6) in the weak field approximation. Suppose that we have a small deviation from the background as:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu},$$

where $\bar{g}_{\alpha\beta}$ is the background metric and $h_{\alpha\beta}$ is very small compared to $\bar{g}_{\alpha\beta}$ at every point. It is shown that the linear part of Christoffel connections, Riemann tensor and Ricci tensor become as follows [14]:

$$(\Gamma_{\mu\rho}^\alpha)_L = \frac{1}{2}\bar{g}^{\alpha\lambda}(\bar{\nabla}_\mu h_{\rho\lambda} + \bar{\nabla}_\rho h_{\mu\lambda} - \bar{\nabla}_\lambda h_{\mu\rho}), \quad (2.8)$$

$$(R^\mu{}_{\nu\rho\sigma})_L = \frac{1}{2}\left(\bar{\nabla}_\rho\bar{\nabla}_\sigma h_\nu^\mu + \bar{\nabla}_\rho\bar{\nabla}_\nu h_\sigma^\mu - \bar{\nabla}_\rho\bar{\nabla}^\mu h_{\sigma\nu} - \bar{\nabla}_\sigma\bar{\nabla}_\rho h_\nu^\mu - \bar{\nabla}_\sigma\bar{\nabla}_\nu h_\rho^\mu + \bar{\nabla}_\sigma\bar{\nabla}^\mu h_{\rho\nu}\right), \quad (2.9)$$

$$(R_{\nu\sigma})_L = \frac{1}{2}\left(\bar{\nabla}_\mu\bar{\nabla}_\sigma h_\nu^\mu + \bar{\nabla}_\mu\bar{\nabla}_\nu h_\sigma^\mu - \bar{\square}h_{\sigma\nu} - \bar{\nabla}_\sigma\bar{\nabla}_\nu h\right), \quad (2.10)$$

$$R_L = \bar{g}^{\alpha\beta}(R_{\alpha\beta})_L - \bar{R}^{\alpha\beta}h_{\alpha\beta}, \quad (2.11)$$

$$u^\mu = \bar{u}^\mu + (\Gamma_{0\lambda}^\mu)_L x^\lambda. \quad (2.12)$$

With these relations in hand, let us consider the linear form Eq. (2.6) in generic background without any external force. First order expansion of the left hand side of (2.6) becomes:

$$m(u^\nu \nabla_\nu u^\mu)_L = m\bar{u}^\nu \bar{\nabla}_\nu \left((\Gamma_{0\lambda}^\mu)_L x^\lambda \right) + m\bar{u}^\nu (\Gamma_{\lambda\nu}^\mu)_L \bar{u}^\lambda + m(\bar{\nabla}_\nu \bar{u}^\mu)(\Gamma_{0\lambda}^\nu)_L x^\lambda. \quad (2.13)$$

At the right hand side we have $R_\nu^\mu u^\nu$ and $u^\mu R_{\nu\gamma} u^\nu u^\gamma$, where up to first order after doing some calculation, can be written as follow:

$$\begin{aligned} (R_\nu^\mu u^\nu)_L &= \bar{g}^{\mu\alpha} (R_{\alpha\nu})_L \bar{u}^\nu - h^{\mu\alpha} \bar{R}_{\alpha\nu} \bar{u}^\nu + \bar{R}_\nu^\mu (\Gamma_{0\lambda}^\nu)_L x^\lambda, \\ (u^\mu R_{\nu\gamma} u^\nu u^\gamma)_L &= \bar{u}^\mu \bar{u}^\nu \bar{u}^\gamma (R_{\nu\gamma})_L + \bar{R}_{\nu\gamma} \bar{u}^\nu \bar{u}^\gamma (\Gamma_{0\lambda}^\mu)_L x^\lambda + \bar{R}_{\nu\gamma} \bar{u}^\gamma \bar{u}^\mu (\Gamma_{0\lambda}^\nu)_L x^\lambda \\ &\quad + \bar{u}^\mu \bar{R}_{\nu\gamma} \bar{u}^\nu (\Gamma_{0\lambda}^\gamma)_L x^\lambda, \end{aligned} \quad (2.14)$$

Therefore at the linear level without any external forces, equation (2.6) turns to:

$$\begin{aligned} m \left\{ \bar{u}^\nu \bar{\nabla}_\nu \left((\Gamma_{0\lambda}^\mu)_L x^\lambda \right) + \bar{u}^\nu (\Gamma_{\lambda\nu}^\mu)_L \bar{u}^\lambda + (\bar{\nabla}_\nu \bar{u}^\mu)(\Gamma_{0\lambda}^\nu)_L x^\lambda \right\} &= \frac{q^2}{6} \left\{ \bar{g}^{\mu\alpha} (R_{\alpha\nu})_L \bar{u}^\nu - h^{\mu\alpha} \bar{R}_{\alpha\nu} \bar{u}^\nu \right. \\ &\quad + \bar{R}_\nu^\mu (\Gamma_{0\lambda}^\nu)_L x^\lambda + \bar{u}^\mu \bar{u}^\nu \bar{u}^\gamma (R_{\nu\gamma})_L + \bar{R}_{\nu\gamma} \bar{u}^\nu \bar{u}^\gamma (\Gamma_{0\lambda}^\mu)_L x^\lambda + \bar{R}_{\nu\gamma} \bar{u}^\gamma \bar{u}^\mu (\Gamma_{0\lambda}^\nu)_L x^\lambda \\ &\quad \left. + \bar{u}^\mu \bar{R}_{\nu\gamma} \bar{u}^\nu (\Gamma_{0\lambda}^\gamma)_L x^\lambda \right\} + (f_{self}^\mu)_L. \end{aligned} \quad (2.15)$$

Where the linear form of the self-force is as follows:

$$(f_{self}^\mu)_L = q^2 \left(2\bar{u}^\mu \bar{u}^\alpha h_{\nu\alpha} + \bar{g}_{\nu\alpha} \bar{u}^\mu (\Gamma_{0\lambda}^\alpha)_L x^\lambda + \bar{g}_{\nu\alpha} \bar{u}^\alpha (\Gamma_{0\lambda}^\mu)_L x^\lambda - h_\nu^\mu \right) \int_{-\infty}^{\tau} \partial^\nu G_R(x, x') d\tau'. \quad (2.16)$$

The equation (2.15) can be considered as the linear form of (2.6) in generic background. What one shall do is computing the linear Christoffel connections, Riemann and Ricci tensors and also the proper Green's functions.

2.2 Flat background case:

Suppose in flat background we have a small perturbation, $h_{\mu\nu}$, where

$$h_{\mu\nu} = \begin{cases} h_{00} = -\phi(r), \\ h_{ij} = -\phi(r), \quad i = j \end{cases} \quad (2.17)$$

where $\phi(r)$ is a time-independent scalar function of r . And also 4-velocity in flat background has been used as $\bar{u}^\mu = \gamma(1, \vec{v})$, where γ is the usual relativity factor, but in this case we consider it as $\bar{u}^\mu = (1, 0, 0, 0)$, namely for the comoving observer while relates to the other inertial observers by the Lorentz transformation. Note that hereafter we omit the bar over the velocities since they are all written in the background. It is easy to calculate that

$$\begin{aligned} (\Gamma_{0\lambda}^\mu)_L x^\lambda &= \frac{1}{2} \eta^{\mu\alpha} (r \partial_r h_{0\alpha} - t \partial_\alpha h_{00}), \\ (\Gamma_{0\lambda}^\nu)_L x^\lambda \partial_\nu u^\mu &= -\frac{1}{2} \partial_r h_{00} (r \partial_0 + t \partial_r) u^\mu, \\ (R_\nu^\mu u^\nu)_L &= \eta^{\mu\alpha} (R_{\alpha\nu})_L u^\nu \end{aligned} \quad (2.18)$$

After doing some calculation we obtain

$$\begin{aligned} \partial_0 (r \partial_r h_0^\mu - t \partial^\mu h_{00}) - \partial^\mu h_{00} &= -\frac{q^2}{6m} \left(\square h_0^\mu + u^\mu \square h_{00} \right) + \frac{q^2}{m} \left(4u^\mu h_{0\nu} + u^\mu (r \partial_r h_{0\nu} - t \partial_\nu h_{00}) \right. \\ &\quad \left. + u_\nu (r \partial_r h_0^\mu - t \partial^\mu h_{00}) - 2h_\nu^\mu \right) \int_{-\infty}^{\tau} \partial^\nu G_R(x, x') d\tau', \end{aligned} \quad (2.19)$$

we have maintained u^μ in order to keep the covariance. The equation (2.19) can be simplified as:

- for $\mu = 0$, we obtain:

$$\dot{r}\partial_r\phi = -\frac{q^2}{m}\left((6\phi + 2r\partial_r\phi)\partial_0\mathcal{A} + t\partial_r\phi\partial_r\mathcal{A}\right), \quad (2.20)$$

- for $\mu = 1$, we have:

$$2\partial_r\phi = \frac{q^2}{m}\left(t\partial_r\phi\partial_0\mathcal{A} + 2\phi\partial_r\mathcal{A}\right), \quad (2.21)$$

Where $\mathcal{A} \equiv \int_{-\infty}^{\tau} G_R(x, x') d\tau'$.

From (2.20) and (2.21) we obtain:

$$2\dot{r}\partial_r\mathcal{A} = \frac{q^2}{m}\left[\left(6t - \frac{12m}{q^2} - 4r\partial_r\mathcal{A}\right)\partial_0\mathcal{A} - 2(\partial_r\mathcal{A})^2\right]. \quad (2.22)$$

What is interesting here, can be stated as follows:

when we consider the perturbation in the flat background, the trajectory of the particle is independent of the perturbation factor, at least for our case when ϕ is supposed to be a function of r only.

Now let us recall the Green's function in the flat space-time.

2.3 Green's functions:

Green's functions are actually an integral kernel that can be used to solve an inhomogeneous differential equation with boundary conditions in which for a complete set of modes they are defined by

$$G(x, x') = \sum_n \phi_n(x)^* \phi_n(x').$$

In quantum mechanics, the vacuum expectation values of various products of free field operators can be identified with various Green's functions of the wave equation. For example, the Pauli-Jordan or Schwinger function is defined by the vacuum expectation value of the commutator of fields as follows [15]

$$iG(x, x') = \langle 0 | (\phi(x)\phi(x') - \phi(x')\phi(x)) | 0 \rangle. \quad (2.23)$$

One can define retarded and advance Green's functions, which are relevant to our work, as

$$\begin{aligned} G_R(x, x') &= -\theta(t - t')G(x, x'), \\ G_A(x, x') &= \theta(t' - t)G(x, x'), \end{aligned} \quad (2.24)$$

where

$$\theta(t) = \begin{cases} 1, & t > 0; \\ 0, & t < 0. \end{cases} \quad (2.25)$$

Note that as mentioned all these forms satisfy in field equation

$$(\square - m^2)G_{A,R}(x, x') = \delta^4(x, x').$$

In flat space the proper retarded Green's function is given by

$$G_R(\vec{x}, t; \vec{x}', t') = \frac{\delta(t - t' + |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|}, \quad (2.26)$$

3 de Sitter Space-time

Recent observational data are strongly in favor of a positive acceleration of our expanding universe [5]. In the first approximation, the background space-time might be considered as de Sitter space-time. The de Sitter space-time is a solution of the vacuum Einstein equation with a positive cosmological constant Λ ,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 0. \quad (3.27)$$

The de Sitter space-time is the unique maximally symmetric curved space-time with ten Killing vectors (the same as Minkowski space-time) and locally characterized by the condition [16]:

$$\bar{R}_{\mu\nu\lambda\rho} = \frac{2\Lambda}{(D-1)(D-2)}(\bar{g}_{\mu\lambda}\bar{g}_{\nu\rho} - \bar{g}_{\mu\rho}\bar{g}_{\nu\lambda}). \quad (3.28)$$

$\bar{R}_{\mu\nu\lambda\rho}$ is the Riemann curvature tensor. Using the relations $\bar{R}_{\mu\nu} = \bar{R}^\lambda_{\mu\lambda\nu}$, $\bar{R} = \bar{g}_{\mu\nu}\bar{R}^{\mu\nu}$ we obtain:

$$\begin{aligned} \bar{R}_{\mu\nu} &= \frac{2\Lambda}{D-2}\bar{g}_{\mu\nu}, \\ \bar{R} &= \frac{2D}{D-2}\Lambda, \end{aligned} \quad (3.29)$$

where R is the Ricci scalar and D is the dimension of the space-time. The metric in de Sitter space-time is defined by

$$ds^2 = \bar{g}_{\mu\nu}dX^\mu dX^\nu, \quad \mu, \nu = 1, 2, \dots, D, \quad (3.30)$$

in which X^μ belongs to the intrinsic coordinates of this four dimensional hyperbolic space-time. Note that for the sake of simplicity, only in this section, the Greek indexes run from 1. It means that de Sitter space-time can be represented by a hyperboloid, embedded in a $D+1$ -dimensional flat space (Ambient space notation), with one constraint:

$$X_H = \{x \in R^{D+1}; x^2 = \eta_{ab}x^a x^b = H^{-2}\}, \quad a, b = 1, 2, \dots, D+1, \quad (3.31)$$

for which the metric is given by:

$$ds^2 = \eta_{ab}dx^a dx^b, \quad \eta_{ab} = \text{diag}\{-1, 1, 1, \dots, (D+1)\}, \quad (3.32)$$

where H^{-1} is the minimum radius of the hyperboloid and H is the Hubble parameter, where we have

$$\Lambda = \frac{(D-2)(D-1)}{2}H^2. \quad (3.33)$$

In this work we take $D = 4$.

Generally, one can define four classes of coordinate systems in de Sitter space-time, Global, Conformal, Flat and Static (for more review see [17]). Here, we recall two of them, Flat and Static coordinates:

i) Flat coordinates: This metric is defined by:

$$ds^2 = \eta_{ab}dx^a dx^b = \bar{g}_{\mu\nu}dX^\mu dX^\nu = dt^2 + e^{-2Ht}dX_i^2, \quad i = 2, 3, 4 \quad (3.34)$$

where

$$\begin{aligned} x_1 &= H^{-1} \sinh Ht - \frac{1}{2}H e^{-Ht} X_i X_i, \\ x_i &= e^{-Ht} X_i, \\ x_5 &= H^{-1} \cosh Ht - \frac{1}{2}H e^{-Ht} X_i X_i. \end{aligned} \quad (3.35)$$

This metric due to its simplicity and similarity to the Minkowski space-time, is more applicable in literature, note that the spacial part is flat.

ii) Static coordinates: The metric is defined by

$$ds^2 = -(1 - H^2 r^2)dt^2 + (1 - H^2 r^2)^{-1}dr^2 + r^2 d\Omega^2, \quad (3.36)$$

where

$$\begin{aligned} x_1 &= H\sqrt{1 - H^2 r^2} \sinh Ht, \\ x_2 &= r \sin \theta \cos \varphi, \\ x_3 &= r \sin \theta \sin \varphi, \\ x_4 &= r \cos \theta, \\ x_5 &= H\sqrt{1 - H^2 r^2} \cosh Ht. \end{aligned} \quad (3.37)$$

Only in this coordinate system $\frac{\partial}{\partial t}$ is a time-like Killing vector, and due to this property it is frequently used by authors. We leave more details to the relevant references.

It is worth to mention that although de Sitter space-time has a non-vanishing Ricci scalar, however in local consideration and in the mentioned coordinate, Eq. (2.5) is still applicable. Therefore, the modified equation of motion can be considered by replacing $f_{total} \equiv ma$ in the total force and $f_{self} = \frac{2}{3}q^2\ddot{v}$. It is easy to show that the following relation holds between f_{self} and f_{ext} (see appendix):

$$f_{self} = \sum_{n=1}^{\infty} \alpha^n \left(\frac{d^n}{dt^n} \right) f_{ext}, \quad (3.38)$$

where $\alpha \equiv \frac{2}{3} \frac{q^2}{mc^3}$. Here, we consider only the first derivative, because $\frac{q^2}{m}$ is considered to be very small. So one obtains:

$$f_{total} = \sum_{n=0}^{\infty} \alpha^n \left(\frac{d^n}{dt^n} \right) f_{ext}. \quad (3.39)$$

3.1 Geodesics:

The aim of this section is to obtain the geodesic equations in de Sitter space-time. These equations can be deduced from

$$\frac{d^2 X^\mu}{d\tau^2} + \Gamma_{\nu\rho}^\mu \frac{dX^\nu}{d\tau} \frac{dX^\rho}{d\tau} = 0, \quad (3.40)$$

where τ is the affine parameter that here is considered as the proper time. Using the metric (3.34) in Eq. (3.40) gives us the geodesic equations:

$$\begin{aligned} \frac{d^2}{d\tau^2} t(\tau) &= -He^{2Ht} \sum_{i=1}^3 \left(\frac{d}{d\tau} X_i(\tau) \right)^2, \\ \frac{d^2}{d\tau^2} X_i(\tau) &= -2H \left(\frac{d}{d\tau} t(\tau) \right) \frac{d}{d\tau} X_i(\tau). \end{aligned} \quad (3.41)$$

Let us consider only the $i = 1$ case. From the geodesic equations, we can obtain two differential equations:

$$\frac{d}{d\tau} X(\tau) = e^{-2Ht(\tau)} \quad (3.42)$$

$$\frac{d^2}{d\tau^2} t(\tau) = -3He^{-2Ht(\tau)}. \quad (3.43)$$

After doing some algebra we obtain (appendix)

$$\dot{v} = -2Hv \left(1 - \frac{3}{2}v^2 e^{2Ht} \right), \quad (3.44)$$

where $v = \dot{X}$. Due to the geodesic equations, any rest mass in a local frame, is accelerating, and then the four-acceleration a^μ is non-zero. This was shown in equation (3.44), which confirms that the massive object is being pushed off its geodesics, and therefore is accelerating. Note that if we suppose $v \ll 1$, then we will obtain $v = v_0 e^{-2Ht}$; this formula notifies the motion of an object in a fluid. From Eq. (3.44), we obtain:

$$v = \frac{e^{-Ht}}{\sqrt{3 + e^{2Ht} \left(\frac{1}{v_0^2} - 3 \right)}}. \quad (3.45)$$

Thus the following condition on the initial velocity must be satisfied:

$$v_0 < \frac{1}{\sqrt{3(1 - e^{-2Ht})}}.$$

In the flat limit case ($H \rightarrow 0$), we have no change in particle's velocity. It means that the curvature change the velocity as we expected. The case $v_0 = \frac{1}{\sqrt{3}}$, is somehow interesting since in the weak field limit, a particle moving radially in Schwarzschild background, neither accelerates nor decelerates¹ [18]. With this initial velocity, we obtain

$$v = \frac{e^{-Ht}}{\sqrt{3}}, \quad (3.46)$$

¹ $v = \frac{1}{\sqrt{3}}$ is also the speed of sound in an ultra-relativistic medium, note that we have taken $c = 1$.

on the other hand for a particle that initiated its motion from the early universe, we can write $H_0 T \approx 1$ [19] - where T stands for the present age of our universe - so the initial velocity in the early universe should have been less than 0.7.

Finally one can obtain the following statement for the trajectory of the particle in de Sitter universe:

$$X(t) = \frac{1}{3H} \left(\frac{1}{v_0} - e^{-Ht} \sqrt{3 + (-3 + \frac{1}{v_0^2})e^{2Ht}} \right), \quad (3.47)$$

where $X(0) = 0$.

Local consideration allows us to suppose that we have a gravitational force acting on a freely falling object which is proportional to the velocity and H , (see Eq. 3.44), this force may be considered as an external force caused by the geometry.

3.2 Non-relativistic self-force

Now let us consider the limit $v \ll 1$ in the relation (3.47) i.e. its non-relativistic limit, so we obtain

$$X(t) = \frac{v_0}{2H} (1 - e^{-2Ht}), \quad (3.48)$$

where in the flat limit ($H \rightarrow 0$), it leads to $X(t) = v_0 t$. And also the non-relativistic limit of external force becomes:

$$f_{ext} \equiv f_{NR}^G = -2m_0 H v_0 e^{-2Ht}, \quad (3.49)$$

where f_{NR}^G is the non-relativistic limit of the gravitational force and m_0 is the rest mass (appendix B). Substituting (3.49) in Eq. (3.39), we obtain:

$$X_q(t) = \frac{v_0}{(2H)(1 + 2H\alpha)} (1 - e^{-2Ht}) = \frac{X(t)}{(1 + 2H\alpha)}, \quad (3.50)$$

where $X(t)$ is the trajectory of a charged particle when it undergoes only the gravitational force, however, $X_q(t)$ is its counterpart when we consider the effect of the self-force.

Suppose that, we have two completely identical objects with the same starting points and initial velocities. It is easy to see that because of the radiation reaction of accelerated charged object, at any time we have $X_q(t) < X(t)$, as it was expected.

4 Conclusion

The covariant analysis of the electromagnetic self-force has a quite rich history and dates back to the early work of Abraham in 1933, then Dirac in 1938 used an other method based upon consideration of energy-momentum conservation in flat space-time [2]. The generalization of Dirac's approach to curved space was done by DeWitt and Brehme in 1960 [10]. The other interesting features of the self-force can be found in the literature (for example see [20]).

In the present work, we first linearized the scalar self-force in generic background and then studied the flat background. It was shown if we take the perturbation as (2.17), the trajectory of object will be independent of the perturbation.

On the other hand, in local consideration the particle acts in a way that it is affected by the gravitational force namely the particle moving on its geodesic can be regarded as an object being forced under the gravitational force. Regarding this gravitational force as an external force, we obtained the effect of electromagnetic self-force on particle's trajectory in non-relativistic regime. We considered an extended charged particle, so there were not any singularities in electromagnetic potentials. Therefore it is shown that the trajectory of particle changes from Eq. (3.48) to Eq. (3.50). Note that when we take the limit $H \rightarrow 0$, we regain the classical formula for $X_q(t)$. This means that, we could find the corrections in the particle's trajectory, due to its charge (self-force), in de Sitter space-time in local consideration.

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5 Appendix

5.1 consideration of the equivalence principle:

According to the equivalence principle, we can impose the theory of electromagnetism into general theory of relativity. The equivalence principle tells us that all physical statements are applicable in locally inertial frames (even if totally accelerated) as well as they are in Minkowski space-time. Mathematically, this can be done by taking the covariant derivatives instead of the partial derivatives, when the metric is symmetric. Consequently, the electromagnetic effects, will be the same for a charged object which is at rest.

The self-force is opposite to the direction of acceleration, therefore for a moving object, the external force is acting to neutralize the self-force effects. Also for an object which is at rest, these two forces must be equal, so we should define a mechanical mass (m_{mech}) to distinguish between a resting object and a moving one. Using Eq. (2.3), this can be interpreted as below, where the geodesics themselves, are displaying the four acceleration:

$$m_{mech} \left\{ \frac{d^2 X^\mu}{d\tau^2} + \Gamma_{\nu\rho}^\mu \frac{dX^\nu}{d\tau} \frac{dX^\rho}{d\tau} \right\} = F_{ext}^\mu + F_{self}^\mu. \quad (5.51)$$

If the electromagnetic mass is introduced by:

$$m_{em} a^\mu = -F_{self}^\mu. \quad (5.52)$$

then one can rewrite (5.51) as:

$$(m_{mech} + m_{em}) a^\mu = F_{ext}^\mu. \quad (5.53)$$

And also by letting:

$$m_G \equiv m_{mech} + m_{em}, \quad (5.54)$$

in which m_G is the gravitational mass, for a zero mechanical mass (i.e. for a resting object), we have:

$$F_{ext}^\mu = -F_{self}^\mu. \quad (5.55)$$

and obviously

$$m_G = m_{em} = m_{inertial}, \quad (5.56)$$

where $m_{inertial}$ is the mass for a resting object. However for a moving object, the Newton's second law reads as:

$$\sum_{fields} (F_{self}) + F_{ext} = 0, \quad (5.57)$$

where the self-forces due to the other fields are included. In this work although we worked in the locally flat space, but the object is inserted in a gravitational field, which is described by the general theory of relativity [21].

5.2 Classical external force

From Eq. (3.42) one easily obtains:

$$\frac{dt}{d\tau} = \frac{e^{-2Ht}}{v}, \quad (5.58)$$

where $v = \frac{dX}{dt}$, and also it is easy to show that

$$\frac{d^2}{d\tau^2} X = -2H \frac{e^{-2Ht}}{v} e^{-2Ht}. \quad (5.59)$$

Writing $\frac{d^2}{d\tau^2} = \frac{d^2 t}{d\tau^2} \frac{d}{dt} + (\frac{dt}{d\tau})^2 \frac{d^2}{dt^2}$, the Eq. (5.59) leads to a nonlinear differential equation with respect to v as follows:

$$-3He^{-2Ht}v + \frac{e^{-4Ht}}{v^2}\dot{v} + 2H\frac{e^{-4Ht}}{v} = 0. \quad (5.60)$$

This equation can be simplified to a brief expression as:

$$\dot{v} = -2Hv(1 - \frac{3}{2}v^2e^{2Ht}). \quad (5.61)$$

In order to obtain an expression for the classical geometrical force we start from

$$f_{ext} = \frac{d}{dt}mv, \quad (5.62)$$

where m is relativistic mass. So up to second order in v , we obtain $\dot{m} = m_0v\dot{v}$, so it follows:

$$f_{ext} = m_0v^2\dot{v} + m_0\left(1 + \frac{1}{2}v^2\right)\dot{v}. \quad (5.63)$$

Equation (5.63) together with (5.61) results in: $f_{ext} = -2m_0Hv$. Following calculations are important when we impose the effect of electromagnetic self-force,

$$\dot{v} = a = \frac{f_{ext}}{m} + \alpha\dot{a}. \quad (5.64)$$

Substituting \dot{a} in the right hand side of the Eq. (5.64), results in:

$$a = \frac{f_{ext}}{m} + \alpha\left(\frac{\dot{f}_{ext}}{m} + \alpha\ddot{a}\right), \quad (5.65)$$

and

$$ma = f_{ext} + \alpha\dot{f}_{ext} + \alpha^2\ddot{f}_{ext} + \alpha^3f_{ext}^{(3)} + \dots, \quad (5.66)$$

or we can write:

$$f_{self} = \sum_{n=1}^{\infty} \alpha^n \left(\frac{d^n}{dt^n}\right) f_{ext}. \quad (5.67)$$

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